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Fractional Order and Numerical Solution-Application of Modeling in the Management of Examination Misconducts

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ABSTRACT

The fractional order model that represents the spread of Examination misconduct using compartments of the population of honest students (susceptible), those lightly involved in misconduct (exposed), seriously involved ones (infected), and quitters (removed) is provided. The fractional order derivative is considered in the Caputo sense. To determine the epidemic forecast and persistence, we calculate the reproduction number. Analyzing the stability of this scheme ensures a non-negative and unique solution within the defined domain (0,1). Employing the Laplace-Adomian Decomposition Method aids in estimating the solution for the nonlinear fractional differential equations. Utilizing infinite series helps derive solutions for these equations, ensuring convergence to their precise values. The results obtained align with outcomes from the traditional Differential Transformed Method. Finally, numerical results and an outstanding graphic simulation are presented.

Keywords: Fractional Order, Numerical Solution, Examination Misconduct, Laplace Adomian Decomposition Method

INTRODUCTION

Education stands as a fundamental driver of socioeconomic progress within any nation. Assessing the educational system's performance periodically is accomplished through examinations. Education equips individuals with skills and competencies crucial for the job market, enhancing their abilities. Therefore, examinations play a pivotal role in a country's educational advancement and overall development (Asante-Kyei & Nduro (2014) & Amadi & Opuiyo (2018)).

Acknowledging the significance of quality education is essential for societal progress. Embracing this notion early enables a smoother adaptation to future living and work scenarios. However, Nigeria's educational system faces a persistent challenge - the disturbing trend of examination misconduct (Udofia & Sambo (2021) & Chukwunwogor et al (2013)).

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Examination misconduct encompasses actions by examinees, examiners, administrators, parents, or others that contravene prescribed examination regulations. Cheating during exams, theft of question papers, impersonation, disruptions during exams, obstruction of supervision, forgery of result slips, breach of duty, conspiracy, and tampering or concealing other students' materials constitute examination misconduct (Akunne et al (2021) & Anyamene et al (2015)). Essentially, any dishonest, deceitful, or improper behavior before, during, or after an examination, breaching stipulated rules, falls under examination misconduct. This prevalence in Nigeria significantly undermines the country's educational standards and quality. The situation is concerning as it affects students' academic and social performance adversely.

Extensive literature has explored the causes, issues, and associated factors of examination misconduct ((Asante-Kyei & Nduro (2014), Amadi & Opuiyo (2018), Udofia & Sambo (2021), Chukwunwogor et al (2013)), Akunne et al (2021), Anyamene et al (2015) & Ayoade & Farayola (2020),). However, as far as our knowledge extends, (Abdullahi & Sule (2021)) have developed a mathematical model aimed at managing examination misconduct. Their model categorizes examination misconduct into compartments comprising honest students (susceptible), those marginally involved in misconduct (exposed), heavily involved individuals (infected), and those who have discontinued such behavior (removed).

MODEL DESCRIPTION

A deterministic compartmental modeling strategy was utilized to devise an examination misconduct model, categorizing the total student population $N_h(t)$, at a given time into four subgroups: susceptible students (honest) S_e exposed students (lightly involved in examination misconduct) E_e, infected students (highly involved in examination misconduct) I_e, and removed students (quitters) R_e. Hence the total population is given by:

$$N_h = S_e + E_e + I_e + R_e$$

In this model:

- Susceptible students enter the population consistently through recruitment (admission) at a steady rate Λ_h. Their numbers decline when they interact with either exposed or infected students at a rate $\beta\left(E_e + I_e\right)$, and β defined as the force of infection. These susceptible individuals experience natural death at a specific rate μ.

$$\frac{dS_e}{dt} = \Lambda_h - \left(\beta\left(E_e + I_e\right) + \mu\right)S_e$$

- Exposed students, lightly involved in examination misconduct, increase as susceptible students marginally engage in such behavior (e.g., copying from others) at a rate $\beta\left(E_e + I_e\right)$. Due to the gradual process of quitting misconduct, some highly involved students might revert to the exposed class at a rate γ. The population of exposed students decreases when transitioning to the highly involved class at a rate δ, through natural death at a rate μ, or upon quitting examination misconduct completely at a rate k. Thus,

$$\frac{dE_e}{dt} = \beta(E_e + I_e)S_e + \gamma I_e - (\mu + \delta + k)E_e$$

- Infected students, highly involved in examination misconduct, stem from those lightly involved at a rate δ. They decrease due to natural death at a rate μ and a decline in lightly involved students at a rate γ. Moreover, their population diminishes due to punitive measures like imprisonment or expulsion at a rate μ_0. Yields,

$$\frac{dI_e}{dt} = \delta E_e - (\mu + \mu_0 + \gamma)I_e$$

- Removed students, those who quit involvement in examination misconduct, arise from lightly involved students at a rate k and decrease due to natural death at a rate μ.

$$\frac{dR_e}{dt} = kE_e - \mu R_e$$

This descriptive narrative culminates in a system of differential equations that captures the dynamics and transitions among these distinct student subgroups within the context of examination misconduct.

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - (\beta(E + I) + \mu)S \\
\frac{dE}{dt} &= \beta(E + I)S + \gamma I - (\mu + \delta + k)E \\
\frac{dI}{dt} &= \delta E - (\mu + \mu_0 + \gamma)I \\
\frac{dR}{dt} &= kE - \mu R
\end{aligned} \tag{1}$$

The fusion of the Adomain decomposition technique with the Laplace transform has resulted in a significant approach termed as the Laplace Adomain decomposition method (LADM), first proposed by Adomian in 1980. This method demonstrates effective capabilities in solving various types of differential equations. Our interest in the applications of fractional calculus and LADM has prompted an exploration into the numerical solution of the coronavirus model. Within this model, the Caputo derivative serves as a pivotal differential operator. We have compiled established definitions and findings from existing literature sources (Haq et al., 2017, and Farman, et al., 2018), which will be utilized extensively in this study.

Preliminaries (Basic Theorems)

Definition 1. The Caputo fractional order derivative of a function y on the interval $[0, T]$ is defined by

$$ {}^{c}D_{0+}^{\alpha}y(t)=\frac{1}{\Gamma(n-\alpha)}\int_{0}^{t}(t-s)^{n-\alpha-1}y^{(n)}(s)ds \tag{2}$$

where $n=[\alpha]+1$ *and* $[a]$ *represnt the* Integer part of α

The Riemann-Liouville derivative has drawbacks, notably that the fractional derivative of a constant doesn't yield zero. Consequently, we opt for Caputo's definition due to its suitability for handling initial conditions in fractional differential equations (Farman et al., 2018).

Definition 2 *Laplace* transform of Caputo derivative as

$$L\left\{{}^{c}D^{\alpha}y(t)\right\}=s^{\alpha}y(s)-\sum_{k=0}^{n-1}s^{\alpha-k-1}y^{(k)}(0),\ n-1<\alpha<n,\ n\in N \tag{3}$$

The fractional system of differential equations represents the new differential equation system, and they are provided as follows.

$$\begin{aligned}
D^{\alpha_1}S(t)&=\Lambda-\beta S(t)E(t)-\beta S(t)I(t)-\mu S(t)\\
D^{\alpha_2}E(t)&=\beta S(t)E(t)-\beta S(t)I(t)+\gamma I-(\mu+\delta+k)E(t)\\
D^{\alpha_3}I(t)&=\delta E(t)-(\mu+\mu_0+\gamma)I(t)\\
D^{\alpha_4}R(t)&=kE(t)-\mu R(t)
\end{aligned} \tag{4}$$

Wherever, $\alpha\in(01]$ whilst all other parameters are positive parameters and the given initial conditions are

$$\begin{cases}
S(0)=N_1\\
E(0)=N_2\\
I(0)=N_3\\
R(0)=N_4,
\end{cases} \tag{5}$$

Stability Analysis and Equilibria

Disease-free equilibrium (DFE)

The model (4) has a DFE, obtained by setting the right-hand sides of the equations in (4) to zero, given by

$$\begin{cases} D^{\alpha_1}S(t)=0 \\ D^{\alpha_2}E(t)=0 \\ D^{\alpha_3}I(t)=0 \\ D^{\alpha_4}R(t)=0 \end{cases} \tag{6}$$

$$E_0=\left(S^*,E^*,I^*,R^*\right)=\left(\frac{\Lambda}{\mu},0,0,0\right) \tag{7}$$

Theorem 1: The DFE of E_0 is asymptotically stable (LAS) if $R_0<1$, and unstable if $R_0>1$. Reproductive number (Abdullahi et al., 2015 & Yakubu et al., 2021): The threshold result of this equilibrium is

$$F=\begin{vmatrix} \beta\frac{\Lambda}{\mu} & \beta\frac{\Lambda}{\mu} \\ 0 & 0 \end{vmatrix} \tag{8}$$

$$V=\begin{vmatrix} (\mu+\delta+k) & -\gamma \\ -\delta & (\mu+\mu_0+\gamma) \end{vmatrix} \tag{9}$$

The threshold epidemiological of those involved in examination misconduct, denoted by $R_0=\rho(FV^{-1})$, where ρ denotes the spectral radius, is given by

$$R_0=\frac{\beta\Lambda(\mu+\mu_0+\gamma+\delta)}{\mu\left(\mu^2+(\mu_0+k+\gamma+\delta)\mu+(\delta+k)\mu_0+k\gamma\right)} \tag{10}$$

Endemic equilibrium point (EEP)

Next, conditions for the existence of endemic equilibria for the model (4) are explored. Let

$E_1=\left(S^{**},E^{**},I^{**},R^{**}\right)$

be the arbitrary endemic equilibrium of model (4), in which at least one of the infected components of the model is non-zero. Setting the right-hand sides of the equations in (4) to zero gives the following expressions.

$$
\begin{aligned}
S^{**} &= -\frac{\gamma\delta - kk_1}{\beta(k_1+\delta)}, \\
E^{**} &= -\frac{k_1(\delta\mu\gamma - \mu kk_1 + \Lambda\beta k_1 + \Lambda\delta\beta)}{\beta(k_1\gamma\delta - k_1^2 k + \delta^2\gamma - \delta kk_1)}, \\
I^{**} &= -\frac{\delta(\delta\mu\gamma - \mu kk_1 + \Lambda\beta k_1 + \Lambda\delta\beta)}{\beta(k_1\gamma\delta - k_1^2 k + \delta^2\gamma - \delta kk_1)}, \\
R^{**} &= -\frac{k_1 l(\delta\mu\gamma - \mu kk_1 + \Lambda\beta k_1 + \Lambda\delta\beta)}{\beta(k_1\gamma\delta - k_1^2 k + \delta^2\gamma - \delta kk_1)\mu}
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&where\ k = \mu + \delta + k \\
&k_1 = \mu + \mu_0 + \gamma
\end{aligned}
$$

Furthermore, using Theorem 2 of (Van den Driessche & Watmough (2002) & Sule, & Abdullah (2019)) the following result is established.

Non-negative solution

$$let\ R^4 = \{x \in R^4, x \geq 0\}$$

and

$$x(t) = (S(t), E(t), I(t), R(t))^T$$

Lemma: Let $h(x) \in C[a,b]$ and $D^{\alpha} h(x) \in [a,b]$ for $0 < \alpha \leq 1$. Then

$$h(x) = h(a) + \frac{1}{(\alpha+1)!} D^{\alpha} h(\eta)(x-a)^{\alpha}, \text{with } 0 \leq \eta \leq x \text{ for } x \in (a,b]$$

Theorem 2: There is a unique solution for the initial value problem given by (6), and the solution remains in $R^4, x \geq 0$.(Yakubu et al., 2021 & Mamuda et al., 2017)

Proof: Aim is to show that the domain $R^4, x \geq 0.$ is positively invariant. Since

$$
\begin{aligned}
&D^{\alpha_1} S(t)\big|_{S(t)=0} = \Lambda \geq 0, \\
&D^{\alpha_2} E(t)\big|_{E(t)=0} = \beta S(t) I(t) + \gamma I(t) \geq 0, \\
&D^{\alpha_3} I(t)\big|_{I(t)=0} = \delta E(t) \geq 0, \\
&D^{\alpha_4} R(t)\big|_{R(t)=0} = kE(t) \geq 0.
\end{aligned}
\tag{12}
$$

The non-negative solution satisfied the vector field point into R^4

The Laplace–Adomian Decomposition Method

This section focuses on outlining the overall process of the model (4) incorporating the provided initial conditions. Employing the Caputo fractional derivative system involves applying the Laplace transform to both sides of model (4), resulting in:

$$\begin{aligned} &L\left\{D^{\alpha_1} S(t)\right\} = L\{\Lambda - \beta S(t)E(t) - \beta S(t)I(t) - \mu S(t)\} \\ &L\left\{D^{\alpha_2} E(t)\right\} = L\{\beta S(t)E(t) + \beta S(t)I(t) + \gamma I(t) - (\mu + \delta + k)E(t)\} \\ &L\left\{D^{\alpha_3} I(t)\right\} = \{\delta E(t) - (\mu + \mu_0 + \gamma)I(t)\} \\ &L\left\{D^{\alpha_4} R(t)\right\} = L\{kE(t) - \mu R(t)\} \end{aligned} \tag{13}$$

This implies that

$$\begin{aligned} &S^{\alpha_1} L\left\{D^{\alpha_1} S(t)\right\} - S^{\alpha_1 - 1} S(0) = L\{\Lambda - \beta S(t)E(t) - \beta S(t)I(t) - \mu S(t)\} \\ &S^{\alpha_2} L\left\{D^{\alpha_2} E(t)\right\} - S^{\alpha_2 - 1} E(0) = L\{\beta S(t)E(t) + \beta S(t)I(t) + \gamma I(t) - (\mu + \delta + k)E(t)\} \\ &S^{\alpha_3} L\left\{D^{\alpha_3} I(t)\right\} - S^{\alpha_3 - 1} I(0) = L\{\delta E(t) - (\mu + \mu_0 + \gamma)I(t)\} \\ &S^{\alpha_4} L\left\{D^{\alpha_4} R(t)\right\} - S^{\alpha_4 - 1} R(0) = L\{kE(t) - \mu R(t)\} \end{aligned} \tag{14}$$

Using the initial conditions and taking inverse Laplace transform to system (14), we have

$$\begin{aligned} &S(t) = S(0) = L^{-1}\{\Lambda - \beta S(t)E(t) - \beta S(t)I(t) - \mu S(t)\} \\ &E(t) = E(0) = L^{-1}\{\beta S(t)E(t) + \beta S(t)I(t) + \gamma I(t) - (\mu + \delta + k)E(t)\} \\ &I(t) = I(0) = L^{-1}\{\delta E(t) - (\mu + \mu_0 + \gamma)I(t)\} \\ &R(t) = R(0) = L^{-1}\{kE(t) - \mu R(t)\} \end{aligned} \tag{15}$$

Using the values of initial condition in (15), we get

$$\begin{aligned} &S(t) = N_1 = L^{-1}\{\Lambda - \beta S(t)E(t) - \beta S(t)I(t) - \mu S(t)\} \\ &E(t) = N_2 = L^{-1}\{\beta S(t)E(t) + \beta S(t)I(t) + \gamma I(t) - (\mu + \delta + k)E(t)\} \\ &I(t) = N_3 = L^{-1}\{\delta E(t) - (\mu + \mu_0 + \gamma)I(t)\} \\ &R(t) = N_4 = L^{-1}\{kE(t) - \mu R(t)\} \end{aligned} \tag{16}$$

Assume that the solutions, $S(t), E(t), I(t), R(t)$ in the form of infinite series are given by

$$S(t)=\sum_{n=0}^{\infty}S_n(t)$$
$$E(t)=\sum_{n=0}^{\infty}E_n(t)$$
$$I(t)=\sum_{n=0}^{\infty}I_n(t) \tag{17}$$
$$R(t)=\sum_{n=0}^{\infty}R_n(t)$$

While the nonlinear term involved in the model are $S(t)E(t)$, $S(t)I(t)$and are decomposed as follows

$$S(t)E(t)=\sum_{n=0}^{\infty}A_n$$
$$S(t)I(t)=\sum_{n=0}^{\infty}B_n \tag{18}$$

where A_n *and* B_n are the Adomian polynomials defined as

$$A_n=\frac{1}{\Gamma(n+1)}\frac{d^n}{dt^n}\left[\sum_{k=0}^{\infty}\lambda^k S_k\sum_{k=0}^{\infty}\lambda^k E_k\right]\Bigg|\lambda=0$$
$$B_n=\frac{1}{\Gamma(n+1)}\frac{d^n}{dt^n}\left[\sum_{k=0}^{\infty}\lambda^k S_k\sum_{k=0}^{\infty}\lambda^k I_k\right]\Bigg|\lambda=0 \tag{19}$$

The first three polynomials are given by

$$A_0=S_0(t)E_0(t),$$
$$A_1=S_0(t)E_1(t)+S_1(t)E_0(t)$$
$$A_2=2S_0(t)E_2(t)+2S_1(t)E_1(t)+2S_2(t)E_0(t)$$

$$B_0=S_0(t)I_0(t),$$
$$B_1=S_0(t)I_1(t)+S_1(t)I_0(t) \tag{20}$$
$$B_2=2S_0(t)I_2(t)+2S_1(t)I_1(t)+2S_2(t)I_0(t)$$

Using (17), (19) in model (15), yields

$$L\left\{\sum_{n=0}^{\infty} S_k(t)\right\} = \frac{S_0}{s} + \left[\frac{1}{s^{\alpha}} L\{\Lambda - \beta S(t)E(t) - \beta S(t)I(t) - \mu S(t)\}\right]$$

$$L\left\{\sum_{n=0}^{\infty} E_k(t)\right\} = \frac{E_0}{s} + \left[\frac{1}{s^{\alpha}} L\{\beta S(t)E(t) + \beta S(t)I(t) + \gamma I(t) - (\mu + \delta + k)E(t)\}\right]$$

$$L\left\{\sum_{n=0}^{\infty} I_k(t)\right\} = \frac{I_0}{s} + \left[\frac{1}{s^{\alpha}} L\{\delta E(t) - (\mu + \mu_0 + \gamma)I(t)\}\right] \tag{21}$$

$$L\left\{\sum_{n=0}^{\infty} R_k(t)\right\} = \frac{R_0}{s} + \left[\frac{1}{s^{\alpha}} L\{kE(t) - \mu R(t)\}\right]$$

Matching the two sides of (21) yields the following iterative algorithm:

$$L(S_0) = \frac{N_1}{s}$$

$$L(S_1) = \frac{\Lambda}{s^{\alpha}} - \frac{\beta}{s^{\alpha}} A_0 - \frac{\beta}{s^{\alpha}} B_0 - \frac{\mu}{s^{\alpha}} L\{S_0(t)\}$$

$$L(S_2) = \frac{\Lambda}{s^{\alpha}} - \frac{\beta}{s^{\alpha}} A_1 - \frac{\beta}{s^{\alpha}} B_1 - \frac{\mu}{s^{\alpha}} L\{S_1(t)\} \tag{22}$$

$$\vdots$$

$$L(S_n) = \frac{\Lambda}{s^{\alpha}} - \frac{\beta}{s^{\alpha}} A_n - \frac{\beta}{s^{\alpha}} B_n - \frac{\mu}{s^{\alpha}} L\{S_n(t)\} \quad n \geq 1.$$

$$L(E_0) = \frac{N_2}{s}$$

$$L(E_1) = \frac{\beta}{s^{\alpha}} A_0 - \frac{\beta}{s^{\alpha}} B_0 + \frac{\gamma}{s^{\alpha}} L\{I_1(t)\} - \frac{(\mu + \delta + k)}{s^{\alpha}} L\{E_0(t)\}$$

$$L(E_2) = \frac{\beta}{s^{\alpha}} A_1 - \frac{\beta}{s^{\alpha}} B_1 + \frac{\gamma}{s^{\alpha}} L\{I_2(t)\} - \frac{(\mu + \delta + k)}{s^{\alpha}} L\{E_1(t)\} \tag{23}$$

$$\vdots$$

$$L(E_n) = \frac{\beta}{s^{\alpha}} A_n - \frac{\beta}{s^{\alpha}} B_n + \frac{\gamma}{s^{\alpha}} L\{I_n(t)\} - \frac{(\mu + \delta + k)}{s^{\alpha}} L\{E_n(t)\} \quad n \geq 1.$$

$$L(I_0)=\frac{N_3}{s}$$

$$L(I_1)=\frac{\delta}{s^{\alpha}}L\{E_0(t)\}-\frac{(\mu+\mu_0+\gamma)}{s^{\alpha}}L\{I_0(t)\}$$

$$L(I_2)=\frac{\delta}{s^{\alpha}}L\{E_1(t)\}-\frac{(\mu+\mu_0+\gamma)}{s^{\alpha}}L\{I_1(t)\} \tag{24}$$

$$\vdots$$

$$L(I_n)=\frac{\delta}{s^{\alpha}}L\{E_n(t)\}-\frac{(\mu+\mu_0+\gamma)}{s^{\alpha}}L\{I_n(t)\} \quad n\geq 1.$$

$$L(R_0)=\frac{N_4}{s}$$

$$L(R_1)=\frac{k}{s^{\alpha}}L\{E_0(t)\}-\frac{\mu}{s^{\alpha}}L\{R_0(t)\}$$

$$L(R_2)=\frac{k}{s^{\alpha}}L\{E_1(t)\}-\frac{\mu}{s^{\alpha}}L\{R_1(t)\} \tag{25}$$

$$\vdots$$

$$L(R_n)=\frac{k}{s^{\alpha}}L\{E_n(t)\}-\frac{\mu}{s^{\alpha}}L\{R_n(t)\} \quad n\geq 1.$$

Taking Laplace inverse of (22-25) and considering first three terms at different values of $\alpha = 1, 0.95, 0.85 \text{ and } 0.75$: and using the following values:

Table 1: Description of variables for involved in examination misconducts model

Parameter	**Description**	**Values**
S	Susceptible students	600
E	Exposed student	250
I	Infected students	100
R	Removed students	50

Table 2: Description of parameters for involved in examination misconducts model

Parameter	Description	Estimated value	References
Λ	Recruitment rate of students	0.05	[8]
β	Force of infection	0.23	[8]
μ	Natural death of students	0.0004	[8]
μ_0	Death due to imprisonment and expulsion.	0.05	[8]
γ	Rate of progression from highly involved in examination misconducts	0.04	[8]

δ	Rate of progression from lightly involved in examination misconducts	0.03	[8]
k	Rate of progression from lightly involved in examination misconducts students to quitters	0.09	[8]

From $\alpha = 1$, $(22-25)$ *obtained*

$$\begin{aligned} S(t) &= 600 - 48302.350t - 1.387846490 10^{6} t^{2} + 1.522691 10^{6} t^{3} \\ E(t) &= 250 + 48290.500t + 1.386156060 10^{6} t^{2} + 1.522910000000 10^{8} t^{3} \\ I(t) &= 100 - 6.900t + 724.8543000t^{2} + 13825.42453t^{3} \\ R(t) &= 50 + 9.800t + 965.7904000t^{2} + 18474.54567t^{3} \end{aligned} \tag{26}$$

From $\alpha = 0.95$, $(22-25)$ *obtained*

$$\begin{aligned} S(t) &= 600 - 492942.11550t^{0.95} - 1.387846490 10^{6} t^{1.90} + 1.522691783 10^{8} t^{2.85} \\ E(t) &= 250 + 49282.0221t^{0.95} + 1.386156060 10^{6} t^{1.90} + 1.522910000 10^{8} t^{2.85} \\ I(t) &= 100 - 7.041673892t^{0.95} + 724.8543000t^{1.90} + 13825.42453t^{2.85} \\ R(t) &= 50 + 10.00121799t^{0.95} + 965.7904000t^{1.90} + 18474.54567t^{2.85} \end{aligned} \tag{27}$$

From $\alpha = 0.85$, $(22-25)$ *obtained*

$$\begin{aligned} S(t) &= 600 - 51080.56166t^{0.85} - 1.387846490 10^{6} t^{1.70} + 1.522691783 10^{8} t^{2.55} \\ E(t) &= 250 + 51068.03008t^{0.85} + 1.386156060 10^{6} t^{1.70} + 1.522910000 10^{8} t^{2.55} \\ I(t) &= 100 - 7.296868070t^{0.85} + 724.8543000t^{1.70} + 13825.42453t^{2.55} \\ R(t) &= 50 + 10.36366769t^{0.85} + 965.7904000t^{1.70} + 18474.54567t^{2.55} \end{aligned} \tag{28}$$

From $\alpha = 0.75$, $(22-25)$ *obtained*

$$\begin{aligned} S(t) &= 600 - 52556.10863t^{0.75} - 1.387846490 10^{6} t^{1.50} + 1.522691783 10^{8} t^{2.25} \\ E(t) &= 250 + 52543.21506t^{0.75} + 1.386156060 10^{6} t^{1.50} + 1.522910000 10^{8} t^{2.25} \\ I(t) &= 100 - 7.507650240t^{0.75} + 724.8543000t^{1.50} + 13825.42453t^{2.25} \\ R(t) &= 50 + 10.36366769t^{0.75} + 965.7904000t^{1.50} + 18474.54567t^{2.25} \end{aligned} \tag{29}$$

Differential Transform Method

The following recurrence relation to the system (4) with respected to time (t) is obtained

$$\begin{aligned}
S(k+1) &= \frac{1}{k+1}\left[\Lambda\partial(k) - \beta\sum_{i=0}^{k} S(l)E(k-l) - \beta\sum_{i=0}^{k} S(l)I(k-l) - \mu S(k)\right] \\
E(k+1) &= \frac{1}{k+1}\left[\beta\sum_{i=0}^{k} S(l)E(k-l) + \beta\sum_{i=0}^{k} S(l)I(k-l) + \gamma I(k) - (\mu+\delta+k)E(k)\right] \\
I(k+1) &= \frac{1}{k+1}\left[\delta E(k) - (\mu+\mu_0+\gamma)I(k)\right] \\
R(k+1) &= \frac{1}{k+1}\left[= kE(k) - \mu R(k)\right]
\end{aligned} \tag{30}$$

The inverse differential transform of $S(k)$ is defined as: When t_0 is taken as zero, the given function $y(x)$ is declared by a finite series and above equation can be written in the form $S(t) = \sum_{k=0}^{3} S(k)t^k$

By solving the above equation for $S(k+1), E(k+1), I(k+1)$ *and* $R(k+1)$ up to order 3 we get the function $S(k), E(k), I(k)$ *and* $R(k)$ of respectively

$$\begin{cases}
S(t) = \sum_{k=0}^{3} S(k)t^k \\
E(t) = \sum_{k=0}^{3} E(k)t^k \\
I(t) = \sum_{k=0}^{3} I(k)t^k \\
R(t) = \sum_{k=0}^{3} R(k)t^k
\end{cases} \tag{31}$$

$$\begin{aligned}
S(t) &= 600 - 48302.350t - 1.387846490 10^6 t^2 + 1.522591783 10^8 t^3 \\
E(t) &= 250 + 48290.500t + 1.386156060 10^6 t^2 + 1.522914853 10^8 t^3 \\
I(t) &= 100 - 6.900t + 724.8543000t^2 + 13821.32453t^3 \\
R(t) &= 50 + 9.800t + 965.7904000t^2 + 18473.535677t^3
\end{aligned} \tag{32}$$

NUMERICAL RESULTS

Table 3: Numerical solution of the proposed model using LADM at $\alpha = 1$

Time (week) t	$S(t)$	$E(t)$	$I(t)$	$R(t)$
0	600	250	100	50
0.2	$1.153579096\,10^{6}$	$-1.152973658\,10^{6}$	238.2175682	238.3879814
0.4	$9.504451033\,10^{6}$	$-9.505272830\,10^{6}$	1098.043858	1390.817387
0.6	$3.236213636\,10^{7}$	$-3.236661552\,10^{7}$	3343.099246	4394.066409
0.8	$7.703555566\,10^{7}$	$-7.704696972\,10^{7}$	7637.004111	10134.91324
1.0	$1.508336295\,10^{8}$	$-1.508563034\,10^{8}$	14643.37883	19500.13607

Table 3. Numerical solution of the proposed model using DTM at $\alpha = 1$

Time (week) t	$S(t)$	$E(t)$	$I(t)$	$R(t)$
0	600	250	100	50
0.2	$1.153520868\,10^{6}$	$-1.152999312\,10^{6}$	238.1847682	238.3799014
0.4	$9.503898118\,10^{6}$	$-9.505390978\,10^{6}$	1097.781458	1390.752747
0.6	$3.236017231\,10^{7}$	$-3.236691629\,10^{7}$	3342.213646	4393.848249
0.8	$7.703078400\,10^{7}$	$-7.704756655\,10^{7}$	7634.904911	10134.39612
1.0	$1.508241737\,10^{8}$	$-1.508573330\,10^{8}$	14639.27883	19499.12607

The plots below show the population of each compartment for different values of $\alpha_i (i = 1,2,3,4)$

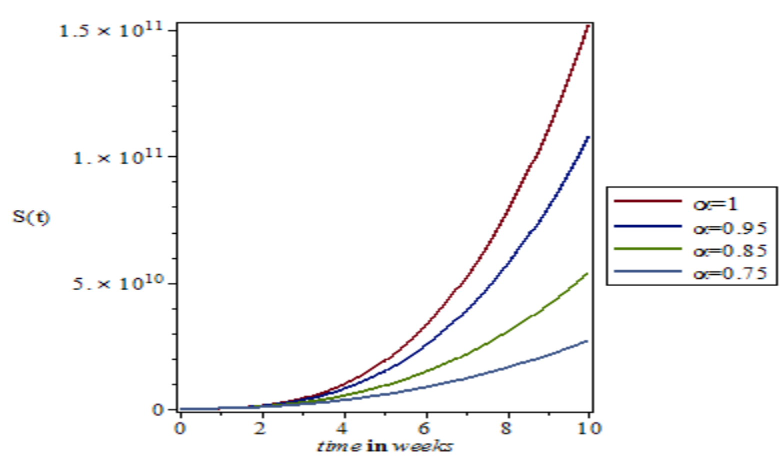

Figure 1: The behavior of the susceptible students

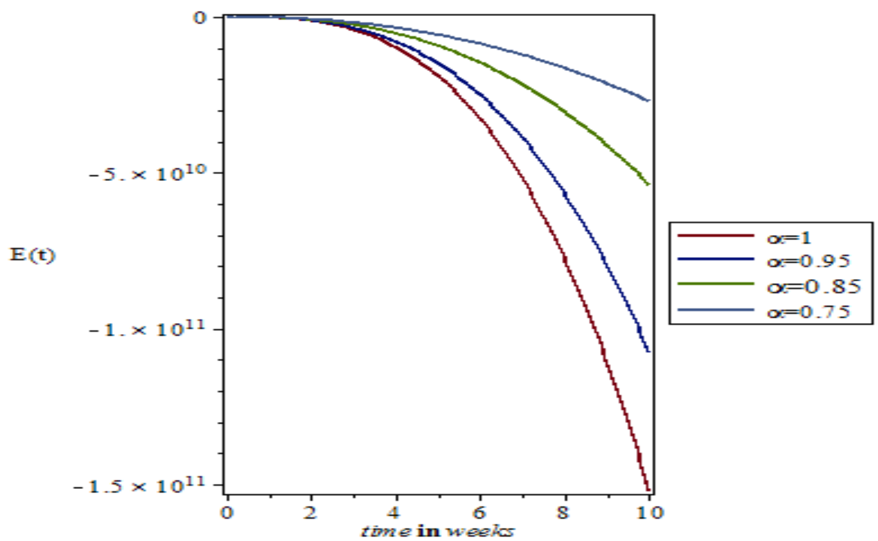

Figure 2: The behavior of the exposed students

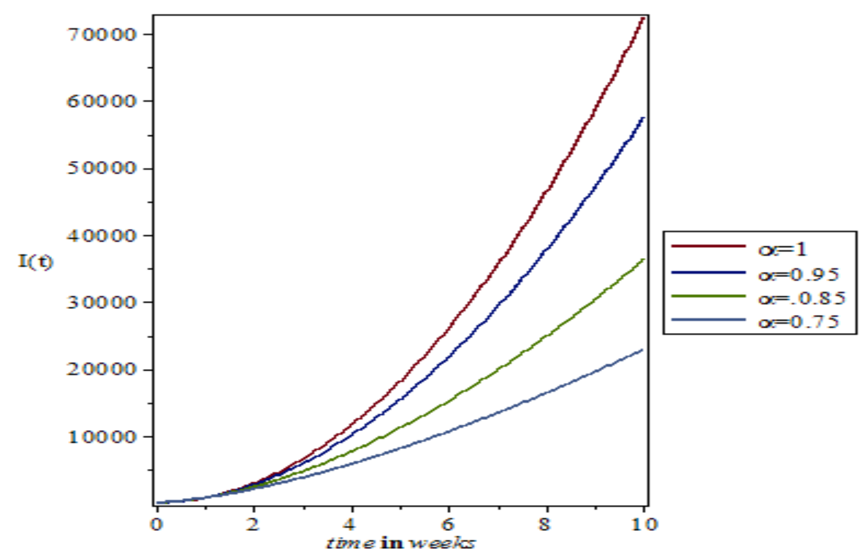

Figure 3: The behavior of the infected students

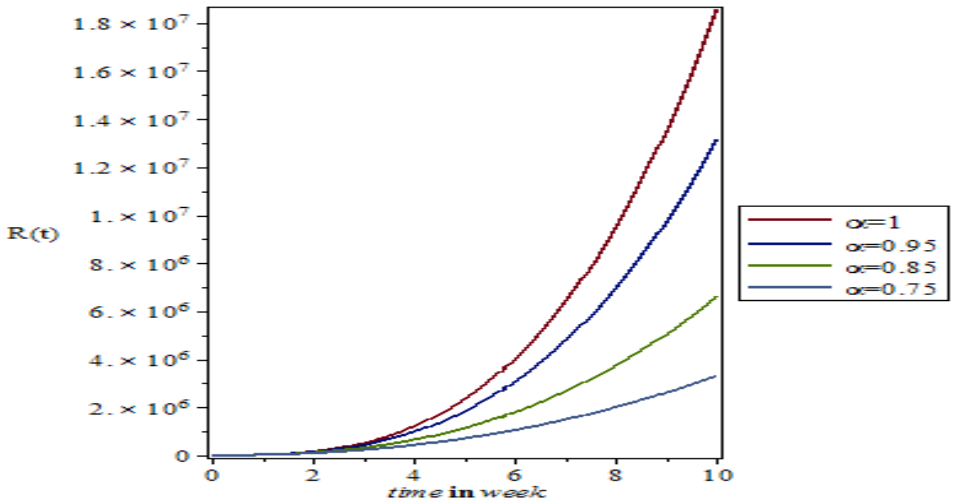

Figure 4: The behavior of the removed (quitters) students

The comparison plots of the LADM and DTM of different compartments

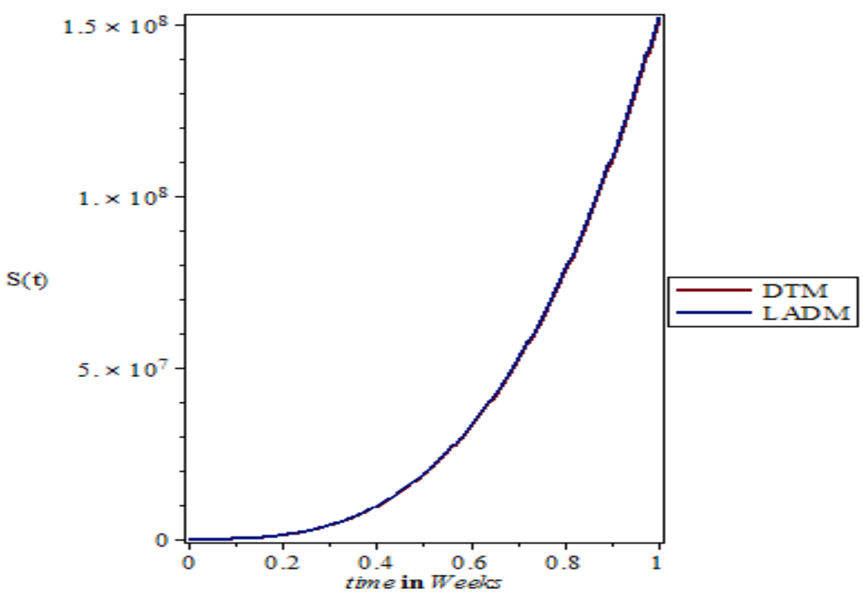

Figure 5: The comparison between the susceptible students using LADM and DTM

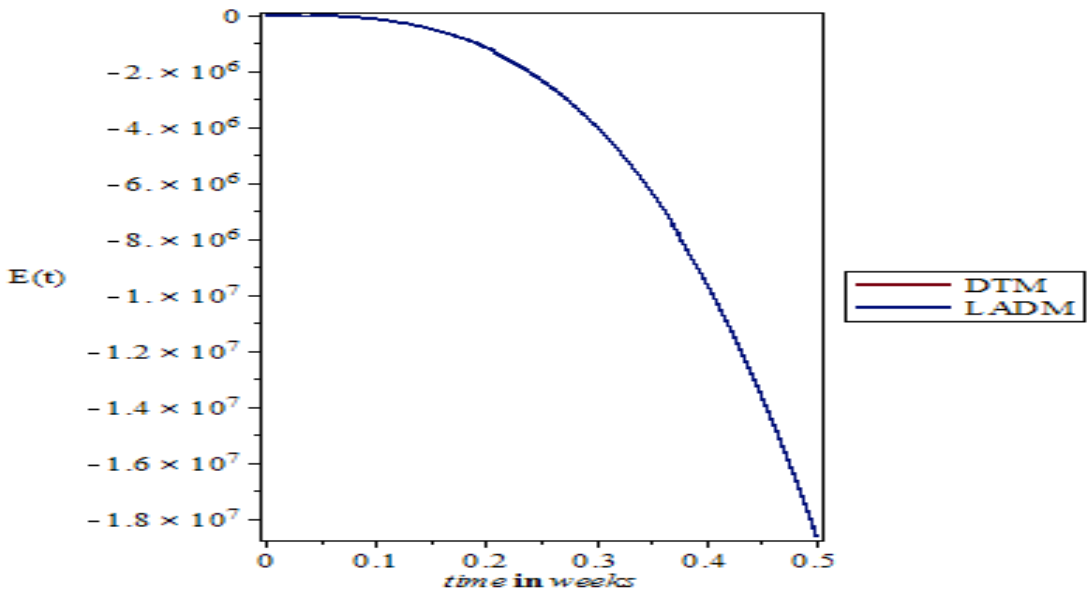

Figure 6: The comparison between the exposed students using LADM and DTM

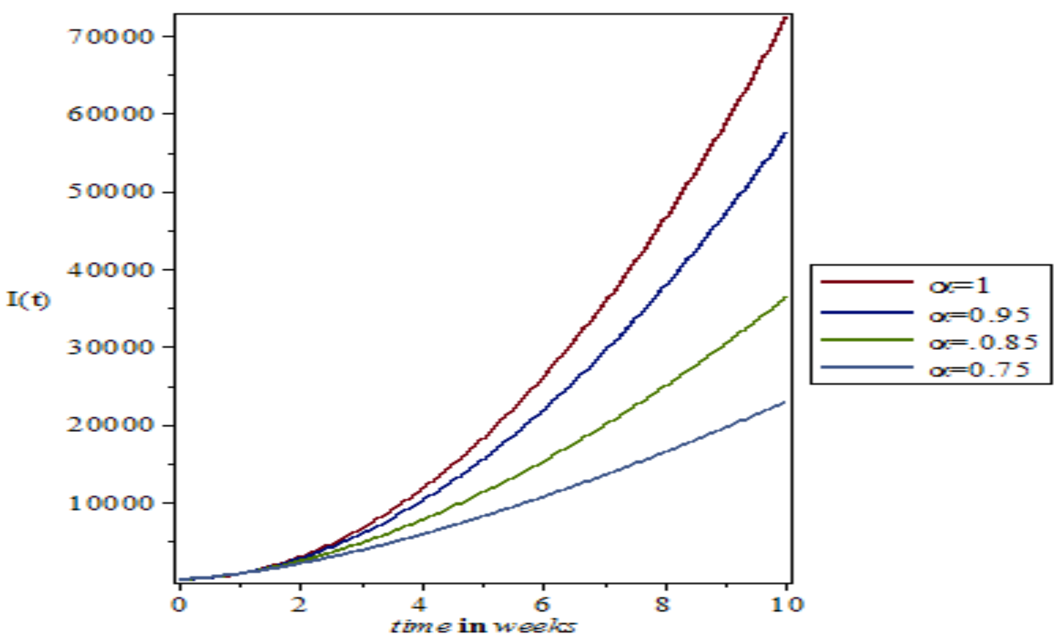

Figure 7: The comparison between the exposed students using LADM and DTM

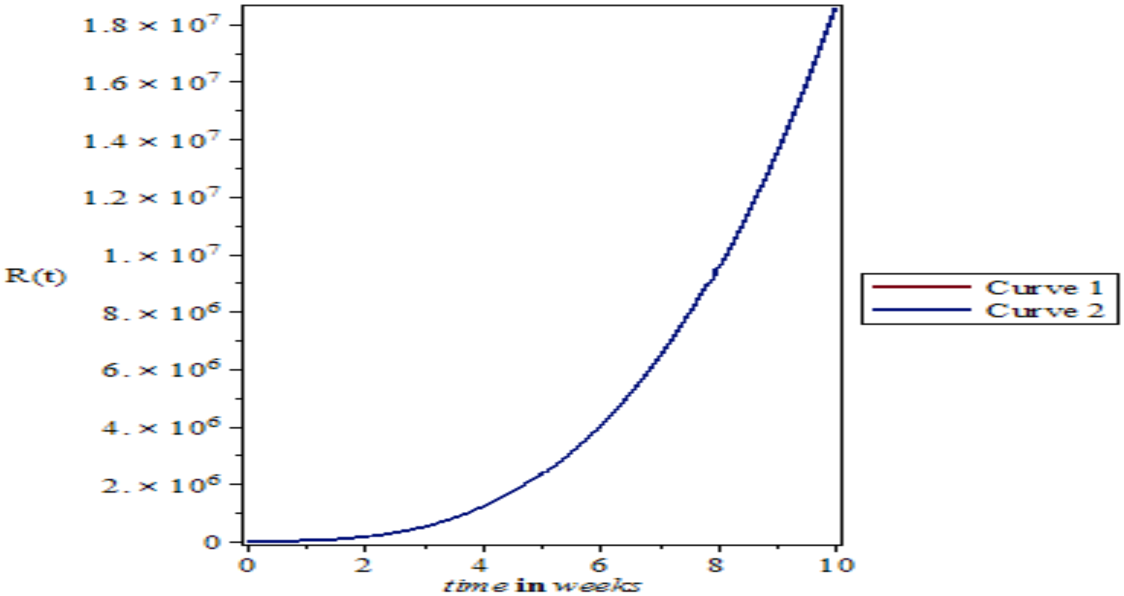

Figure 8: The comparison between the exposed students using LADM and DTM

CONCLUSIONS

This paper employed Caputo-type fractional modeling to study the dynamics of examination misconduct among students. The investigation focused on obtaining the numerical solution using the Laplace-Adomian Decomposition Method (LADM), a powerful tool widely utilized for solving nonlinear models in engineering and applied mathematics. A significant contribution lies in utilizing the Laplace-Adomian Decomposition method to derive the series solution for the fractional model and comparing these outcomes with the classical Differential Transform Method (DTM). The study highlights the strong agreement between the solutions obtained via these methods, demonstrated through tables and graphs. Moreover, the paper illustrates the impact of fractional parameters on our derived solutions using graphical representations.

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